

**Mathematical Modelling and MDDiM Solutions for Fingering Phenomenon during the Nano Silica Flooding**

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**Abstract**

In the primary enhanced oil recovery process (EOR), oil pushes to the earth's surface using the natural pressure of the reservoir. In that process, up to 5% -10% of oil can be extracted. In the secondary oil recovery process, most commonly, water or gases are injected and it allows around 20% -30% of the oil to recover. Recently, researchers found that Nano/biomaterials injection with water (Nano flooding) leads to a maximum of 58% of the estimated oil recovery portion at the secondary stage. This study was carried out to build up a mathematical model to find the saturation of Nano-water ( $S_i$ ) of the fingering phenomenon for an inclined oil layer. The fingering phenomenon (or protuberance) occurs during the secondary oil recovery process when the injection fluid shoots through the porous media at a relative speed. Here we considered silica ( $\text{SiO}_2$  - Silicon dioxide) as the Nano particle. Since Silica is mixable with both oil and water it improves the mobility ratio and increases the injected and native fluids viscosity and recovers an important portion of oil within the secondary EOR process and it is higher than the carbonated-water flooding. We applied the Method of Directly Defining the inverse Mapping (MDDiM) which is a recent method to solve non-linear differential equations to solve this model. MDDiM was first discovered by Liao in 2016 and it was extended by Dewasurendra et al. to solve a system of coupled nonlinear ordinary differential equations. In 2021, Sahabandu et al. further extended this novel technique to solve nonlinear partial differential equations. We obtained second and third-order solutions for  $S_i$  for different incline angles and discussed the convergence of the solutions with the order when squared residual errors are minimum. When the order is higher, we can see that errors are decreasing while converging to a more accurate solution. When  $\alpha=00$ , our MDDiM solutions of second-order for saturations of silica-water are higher than the fourth-order solutions of the variational iteration method of saturation of water. This proves that the Silica-water saturation is higher than the water saturation, and it leads to recovery of more oil by using  $\text{SiO}_2$  together with water.

**Keywords:** *Fingering phenomenon, inclined oil layer, method of directly defining the inverse mapping, nano silica flooding, secondary oil recovery process*

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